Parametric study of hypocycloidal involute gears

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Introduction

Hypocycloidal gear drives with a low difference in numbers of teeth between the ring and planet gears provide a high gear ratio in one stage, compactness and increased load-carrying capacity [1, 2]. Cycloidal and involute tooth profiles are commonly used in hypocycloidal gear drives. This study considers hypocycloidal gears with involute tooth profiles. To avoid potential tooth tip-tip interference in the internal involute gearing with a low difference in numbers of teeth, gear tooth geometry should be non-standard.

The article demonstrates an application of the Direct Gear Design method for parametric analysis of hypocycloidal involute gears.

1. Hypocycloidal Gear Arrangements

Two of the most common hypocycloidal gear arrangements are shown in Figs. 1 and 2.

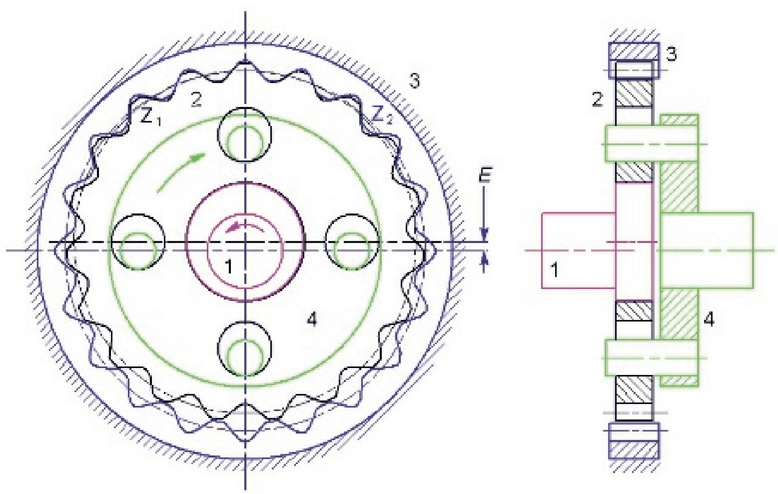


Fig. 1: Planocentric gear arrangement; 1 – input shaft with the eccentric, 2 – planet gear, 3 – ring gear, 4 – carrier connected to output shaft, E – eccentricity, gear center distance.

The first one (Fig. 1), also known as a planocentric gear arrangement, has an eccentric input shaft that drives a planet gear engaged with a stationary ring gear. Torque is transmitted by the holes in the planet gear to the pins pressed into a carrier plate connected to the output shaft. The difference in diameters of the holes and pins is equal to the eccentricity *E*, which is also the gear center distance. The drive gear ratio of this arrangement [3] is

$$u = \frac{z_1}{z_1 - z_2}, \tag{1}$$

where: Z_1 – planet gear number of teeth, Z_2 – ring gear number of teeth.

Since Z_1 is less than Z_2, the gear ratio *u* is negative, and the input and output shafts are rotating in opposite directions.

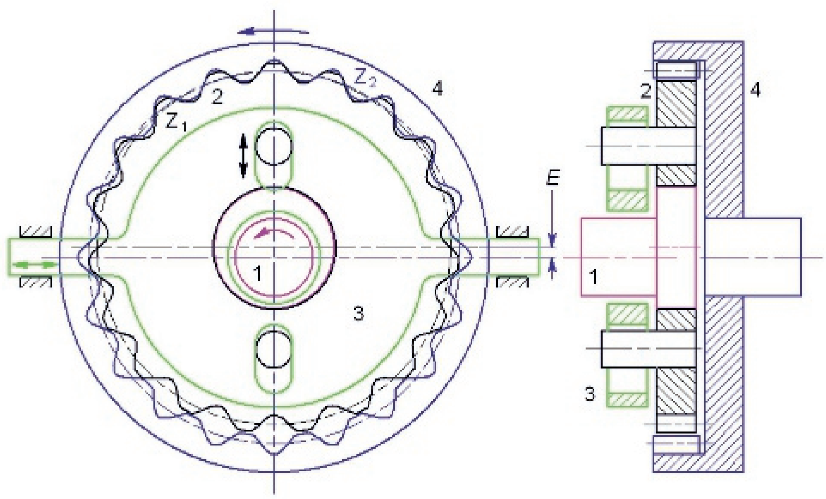


Fig. 2: Arrangement with a wobbling planet gear; 1 – input shaft with the eccentric, 2 – planet gear, 3 – Oldham coupling plate, 4 – ring gear connected to the output shaft, E – eccentricity, gear center distance.

The second arrangement (Fig. 2) with a wobbling planet gear has an Oldham coupling plate with vertical slots. The two pins pressed into the planet gear slide inside slots, preventing rotation of the planet gear. The coupling plate slides horizontally. The wobbling planet gear transmits torque to the rotating ring gear connected to the output shaft. The drive gear ratio of this arrangement [3] is

$$u = \frac{z_2}{z_2 - z_1}. \tag{2}$$

This arrangement always has a positive gear ratio *u*; input and output shafts rotate in the same direction.

2. Interference in Internal Gearing with Low Tooth Number Difference

Tooth tip-tip interference may occur in an internal gearing with a low tooth number difference $Z_2 - Z_1$ and low operating pressure angle α_w. Definition of the tooth tip-tip interference avoidance condition is demonstrated in Fig. 3 and defined by Equations (3 – 5).

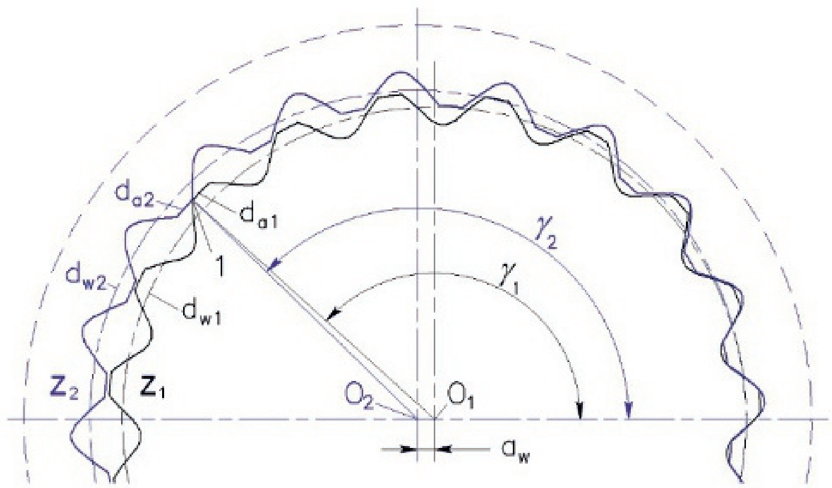


Fig. 3: Tooth tip-tip interference in internal gearing; 1 – interference beginning point, where the planet gear tooth tip touches the ring gear, O_1 and O_2 – centers of the planet and ring gears, a_w – center distance, d_{w1} and d_{w2} – operating pitch diameters of the planet and ring gears, d_{a1} and d_{a2} – tooth tip diameters of the planet and ring gears, γ_1 and γ_2 – angles from the centerline O_1–O_2 to the interference beginning point of the planet and ring gears defined by Equations (3).

$$\gamma_1 = \pi - \arccos\left(\frac{\frac{d_{a1}^2}{4} + {a_w}^2 - \frac{d_{a2}^2}{4}}{d_{a1} a_w}\right), \quad \gamma_2 = \arccos\left(\frac{\frac{d_{a2}^2}{4} + {a_w}^2 - \frac{d_{a1}^2}{4}}{d_{a2} a_w}\right). \tag{3}$$

The tooth tip-tip interference avoidance condition is

$$\Delta = \lambda_1 - u\lambda_2 \geq 0 \ , \tag{4}$$

where:

$$\lambda_{1,2} = \gamma_{1,2} + inv\left(\alpha_{a1,2}\right) - inv\left(\alpha_w\right), \tag{5}$$

α_{a1} and α_{a2} – involute profile angles at the tooth tip diameters of the planet and ring gears (the tooth tip radii are assumed to equal zero); $u = z_2/z_1$ – gear ratio.

This kind of interference can be avoided by increasing the operating pressure angle and reducing the gear tooth height.

The described tooth tip-tip interference avoidance condition does not account for the tooth tip radii of the mating gears. It gives some additional safety margin against this type of interference.

3. Nominal and Effective Contact Ratio in Internal Gearing

The nominal (involute) contact ratio is equal to the length of the contact line *AB* (Fig. 4) divided by the base circle pitch $p_b = \pi d_{b1}/z_1 = \pi d_{b2}/z_2$. For a spur internal involute gear pair [2] it is

$$\varepsilon_\alpha = \frac{z_1}{2\pi}(\tan\alpha_{ae1} - u\tan\alpha_{ae2} + (u-1)\tan\alpha_w)\,, \qquad (6)$$

where: $\alpha_{ae1} = arccos\ (d_{b1}/d_{ae1})$ and $\alpha_{ae2} = arccos\ (d_{b2}/d_{ae2})$ – involute profile angles at the effective tip diameters d_{ae1} and d_{ae2} of the planet and ring gears.

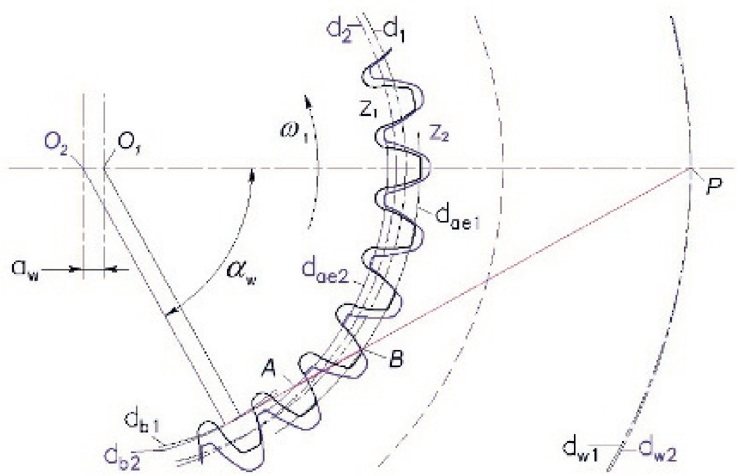


Fig. 4: Approach type internal gear pair; d_{b1} and d_{b2} – base diameters of the planet and ring gears, d_1 and d_2 – reference diameters, d_{w1} and d_{w2} – operating pitch diameters, d_{ae1} and d_{ae2} – effective tip diameters, α_w – operating pressure angle, ω_1 – planet gear rotation direction, *A* and *B* – initial and final point of the involute flank contact, *P* – pitch point.

A smooth transition from one pair of mating spur gears to the next requires a nominal contact ratio $\varepsilon_\alpha > 1.0$. This, as well as the tooth tip-tip interference avoidance condition (Equation 4) for spur internal gears with a low tooth number difference, necessitates using approach type gearing (Fig. 4) when the pitch point *P* is located outside of the contact line *AB*.

The effective contact ratio describes the actual duration of a mating tooth pair contact (Fig. 5). It is defined as the ratio of the tooth engagement angle to the angular pitch. The tooth engagement angle is the gear rotation angle from the initial contact point *C* to the final point *D* of engagement with the mating gear tooth. The effective contact ratio is

$$\varepsilon_{\alpha de} = \frac{\varphi_1}{360^o / z_1} = \frac{\varphi_2}{360^o / z_2}, \tag{7}$$

where: φ_1 and φ_2 - pinion and gear tooth engagement angles,
$360^o/z_1$ and $360^o/z_2$ - pinion and gear angular pitches.

Fig. 5 shows the effective contact ratio definition for conventional type internal gearing when the pitch point *P* is inside of the involute contact line *AB*. This type of internal gears with a low tooth number difference (just one, for example) should have short teeth to avoid tooth tip-tip interference. As a result, a nominal (involute) contact ratio of such internal gears is $\varepsilon_\alpha < 1.0$. Typically, $\varepsilon_\alpha < 1.0$ is unadvisable for spur gears, because it does not provide a smooth transition from one pair of the mating gears to the next, leading to a high transmission error variation, increased noise and vibration, or in some cases complete tooth disengagement. However, in internal gears with a low tooth number difference, mating gear teeth are very close to each other far beyond the involute flank contact line *AB*, where the tooth tip radius of one gear is in contact with the other gear tooth profile near the form diameter. The tooth tip radius should be increased to reduce contact stress and possible wear during tooth tip engagement. This makes the effective contact ratio $\varepsilon_{\alpha e} = 1.0$ even for an unloaded gear pair, compensating for the insufficient nominal (involute) contact ratio $\varepsilon_\alpha < 1.0$. Under load, the effective contact ratio becomes $\varepsilon_{\alpha e} > 1.0$ due to bending and contact tooth deflections [4] and other gear drive component deflections.

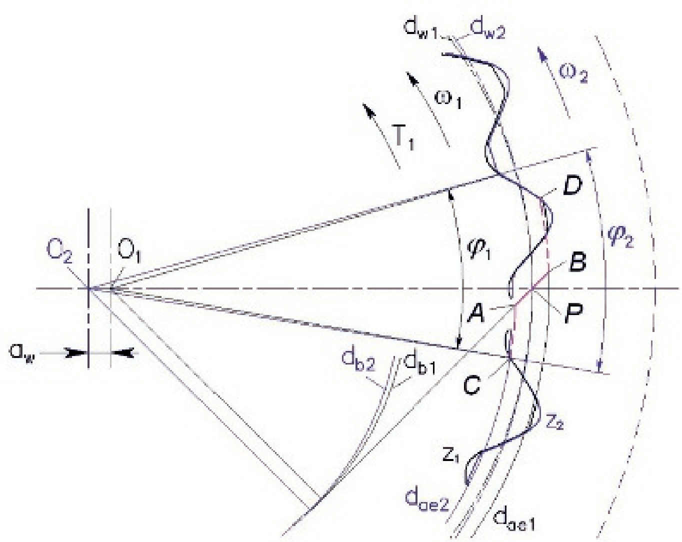


Fig. 5. Effective contact ratio definition; ϕ_1 and ϕ_2 – engagement angles of the planet and ring gears, ω_1 and ω_2 – planet and ring gear rotation directions, T_1 – pinion torque, C – initial point of the tooth contact, A – initial point of the involute flank contact, B – final point of the involute flank contact, D – final point of the tooth contact, P – pitch point.

Transmission error is the angular difference between the actual position of the driven gear and its ideal position (if the gear pair is perfectly conjugated), projected on the line of contact and defined as [5]

$$TE = \frac{d_{b2}}{2}(\theta_2 - \frac{z_1}{z_2}\theta_1), \quad (8)$$

where: θ_1 and θ_2 are the driving and driven gear rotation angles.

4. Hypocycloidal Internal Gearing Design Options

Table 1 presents a comparison between two gear engagement types suitable for hypocycloidal drives: approach type gears with a nominal contact ratio ε_α > 1.0 and conventional type with a nominal contact ratio ε_α < 1.0. It shows that despite the lack of a smooth tooth pair transition, conventional type internal gears with a low tooth number difference have some advantages over approach type gears with a nominal contact ratio ε_α > 1.0, including very low specific sliding of the tooth profiles and significantly lower root and contact stresses.

Table 1: Comparison two gear design options for hypocycloidal drives

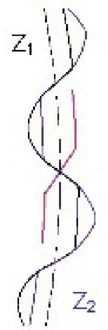
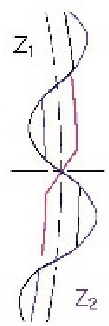
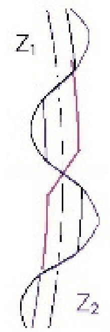
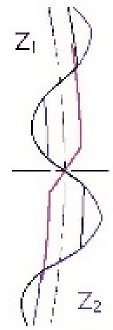
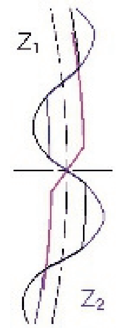
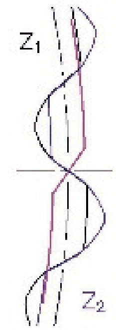
Gear Engagement Type	Approach (Fig. 4)		Conventional (Fig. 5)	
Design Method	Traditional		Direct	
Gear	Planet	Ring	Planet	Ring
Numbers of Teeth	29	30	29	30
Nominal Module, mm	4.000	4.000	4.000	4.000
Nominal Pressure Angle	20°	20°	35°	35°
Reference Diameter (RD), mm	116.000	120.000	116.000	120.000
Base Diameter, mm	109.004	112.763	95.022	98.298
Addendum Modification (X-shift)	**0.0**	**1.0**	**0.0**	**0.0**
Operating Pressure Angle	61.061°	61.061°	35°	35°
Operating Pitch Diameter	225.272	233.040	116.000	120.000
Tooth Tip Diameter, mm	122.000	116.120	118.653	117.546
Root Diameter, mm	105.152	132.968	111.357	125.085
Tooth Thickness at RD, mm	6.283	3.371	6.283	6.283
Tooth Tip Radius, mm	0.40	0.40	0.40	0.40
Root Fillet Profile	Trochoidal	Trochoidal	Optimized	Optimized
Face Width, mm	15.0	15.0	15.0	15.0
Center Distance, mm	**3.884**		**2.000**	
Specific Sliding (tip/root)	**-0.089/-0.242**	**0.195/0.082**	**0.002/-0.002**	**0.002/-0.002**
Nominal Contact Ratio	1.257		0.374	
Effective Contact Ratio	1.257*		1.000*	
Transmission Error Variation, μm	0.0*		2.5*	
Modulus of Elasticity, MPa	207,000	207,000	207,000	207,000
Poisson Ratio	0.3	0.3	0.3	0.3
Torque, Nm	500	517	500	517
Root Stress (FEA), MPa	**340**	**299**	**184(-46%)**	**185(-38%)**
Contact Stress (Hertz), MPa	**199**		**164(-18%)**	

*at zero planet gear torque.

Table 2 presents root and contact stresses, effective contact ratio, and transmission error variations of a conventional engagement internal gear pair for different planet gear torque values.

Table 2

Gear Engagement Type		Conventional					
Design Method		Direct					
Gear		**Planet**			**Ring**		
Numbers of Teeth		29			30		
Module, mm		4.000					
Pressure Angle		35°					
Modulus of Elasticity, MPa		207,000			207,000		
Poisson Ratio		0.3			0.3		
Nominal Contact Ratio		0.374					
Planet Gear Torque, Nm		**0**	**100**	**200**	**300**	**400**	**500**
Root Stress (FEA), MPa	Planet gear	0	36.7	73.4	110	147	184
	Ring Gear	0	37.1	74.2	111	148	185

Contact Stress (Hertz), MPa	0	74.5	107	131	150	164
Effective Contact Ratio	**1.00** 	**1.16** 	**1.30** 	**1.46** 	**1.60** 	**1.72** 
Transmission Error Variation*, μm	**2.5**	**1.8**	**1.4**	**2.0**	**2.8**	**3.6**

*for reference: single pitch deviation of these gears per ISO 1328, grade 5 is ± 6.5 μm

Figs. 6 and 7 show the transmission error and contact ratio charts.

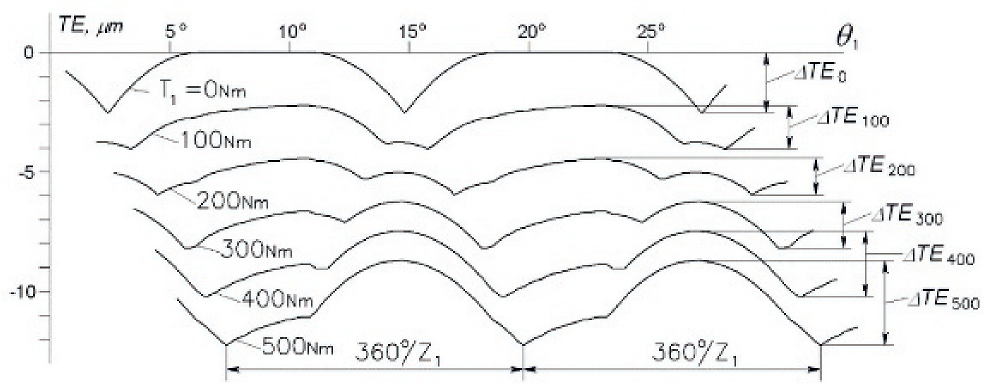


Fig. 6: Transmission error *TE* - planet gear rotation angle θ_1 charts for different planet gear torque T_1 values; ΔTE – transmission error variations.

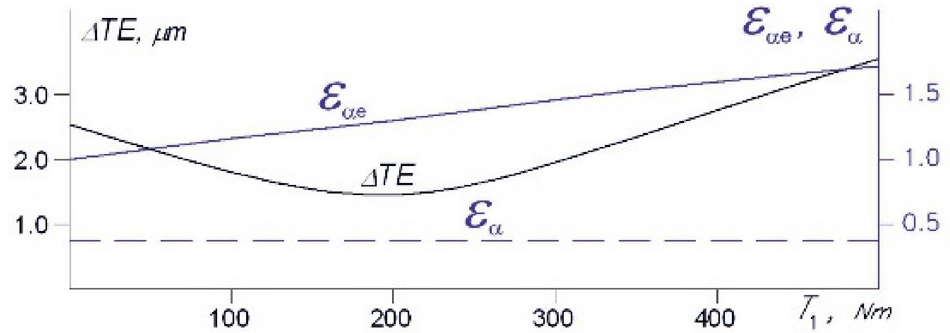
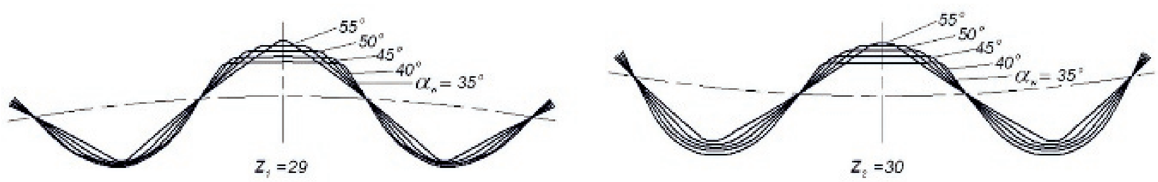


Fig. 7: Transmission error variation ΔTE and nominal ε_{α} and effective $\varepsilon_{\alpha e}$ contact ratios – planet gear torque T_1 charts.

5. Tooth Profiles of Internal Gearing with Low Tooth Number Difference

The tooth geometry of hypocycloidal gears should provide a maximum nominal contact ratio while avoiding the tooth tip-tip interference condition (4). Fig. 8 shows an overlay of planet and ring gear tooth profiles with different pressure angles. The maximum value of the pressure angle is limited by the minimum tooth tip thickness (pointed tooth tip).



a b

Fig. 8: Planet (a) and ring (b) gear tooth profiles with different pressure angles α_W.

The gear ratio of hypocycloidal drives is defined by Equations (1) and (2) in inverse proportion to the tooth number difference between ring and planet gears. The lower the tooth number difference, the higher the gear ratio. Fig. 9 shows an overlay of planet and ring gear tooth profiles with tooth number differences equal to 1, 2, and 3.

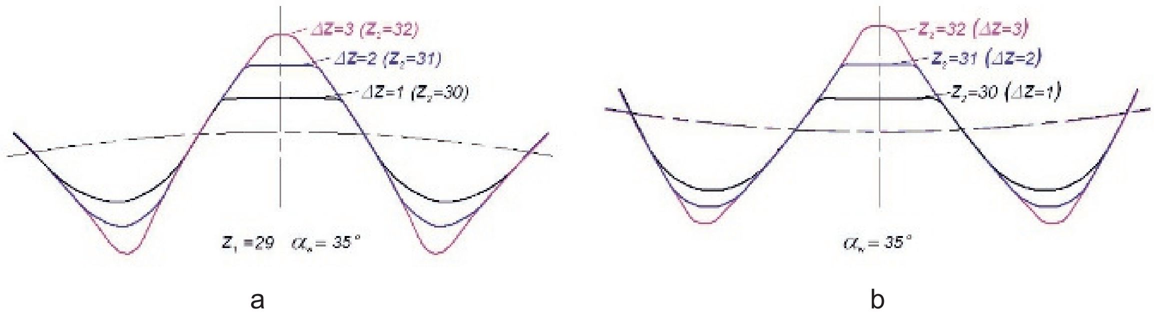


Fig. 9: Planet (a) and ring (b) gear tooth profiles; gear pair Z_1=29, Z_2=30 (ΔZ = 1) – black, gear pair Z_1=29, Z_2=31 (ΔZ = 2) – blue, gear pair Z_1=29, Z_2=32 (ΔZ = 3) – pink.

6. Gear Mesh and Performance Parameters of Internal Gearing with Low Tooth Number Difference

Figs. 10 – 12 present charts of nominal ε_α and effective $\varepsilon_{\alpha e}$ contact ratios, transmission error variation ΔTE, and root $\sigma_{F1,2}$ and contact σ_H stresses as functions of the operating pressure angle α_w for gears with a module m = 4.0 mm, planet number of teeth Z_1 = 29, tooth number differences ΔZ = 1, 2, and 3, planet gear torque T_1 = 500 Nm, Modulus Elasticity E = 207,000 MPa, Poisson Ratio ν = 0.3, face widths of both mating gears $b_{1,2}$ = 15 mm.

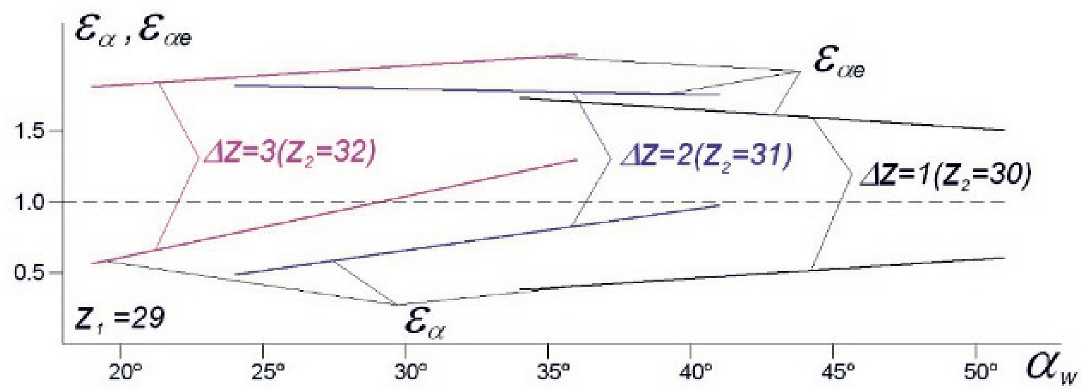


Fig. 10: Nominal ε_α and effective $\varepsilon_{\alpha e}$ contact ratio charts.

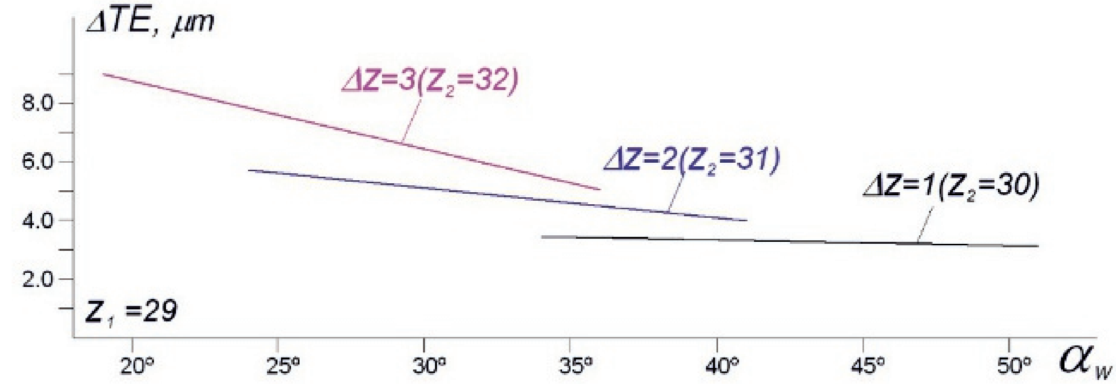


Fig. 11: Transmission error variation ΔTE charts.

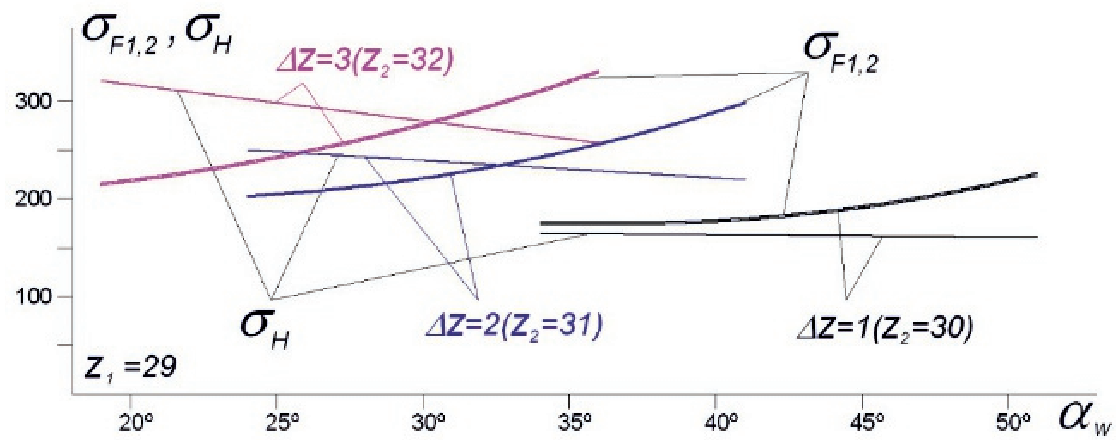


Fig. 12: Root $\sigma_{F1,2}$ and contact σ_H stresses charts.

Besides maximizing the gear ratio, the minimal tooth number difference between the ring and planet gear ΔZ = 1 minimizes transmission error variation and root and contact stresses.

7. Summary

- Hypocycloidal involute drive uses low-tooth-number-difference internal gears to achieve a high gear ratio in one stage.
- Nominal and effective contact ratios are defined for internal gears with a low tooth number difference as well as the tooth tip-tip interference condition.
- Approach and conventional gear engagements are considered for hypocycloidal involute drives. Approach-engagement gears though provide a nominal contact ratio > 1.0, results in high specific sliding, and high root and contact stresses. Conventional engagement gears have minimal specific sliding and low root and contact stresses; however, their

nominal contact ratio < 1.0. Nevertheless, this type of gearing is more beneficial for hypocycloidal involute drives because low-tooth-number-difference internal gears provide an effective gear ratio > 1.0 and relatively low transmission error variation.

- The article presents a study of conventional-engagement hypocycloidal involute gears, constructing tooth profiles with different pressure angles and for tooth number differences equal to 1, 2, and 3. It also defines their effective contact ratio, transmission error variation, and root and contact stresses.

References

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